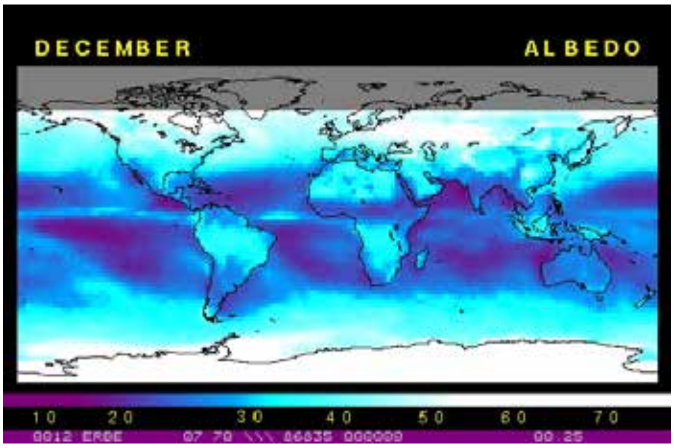
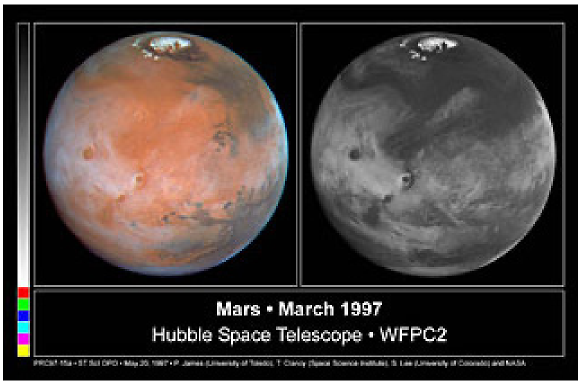
Atmospheric dynamics I

Clouds on the terrestrial planets



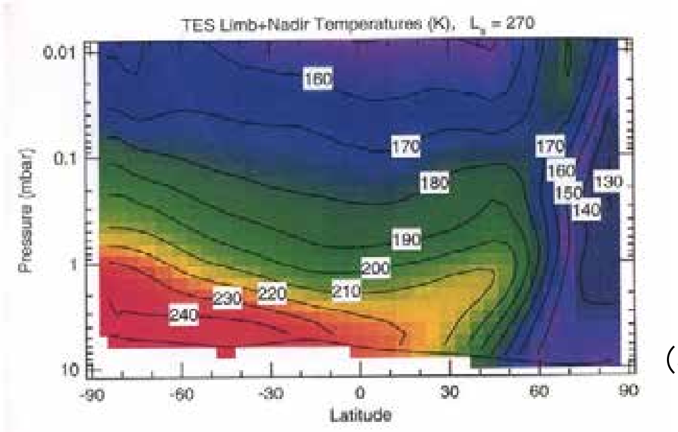
Earth
albedo 0.30



Mars
albedo 0.16

Venus
albedo 0.78

- In one-dimensional model, the atmosphere is assumed to be horizontally uniform
- This is not a good assumption in general



(Smith et al. 2001)

- Large latitudinal/diurnal variation in Mars's atmosphere
- Potentially large horizontal contrast on slow rotators like Venus and tidally-locked exoplanets

Parameter study of the atmospheric circulation of Earth-like planets with general circulation models (GCMs)

Williams (1988)

Meridional stream function

white : anti-clockwise

shade : clockwise

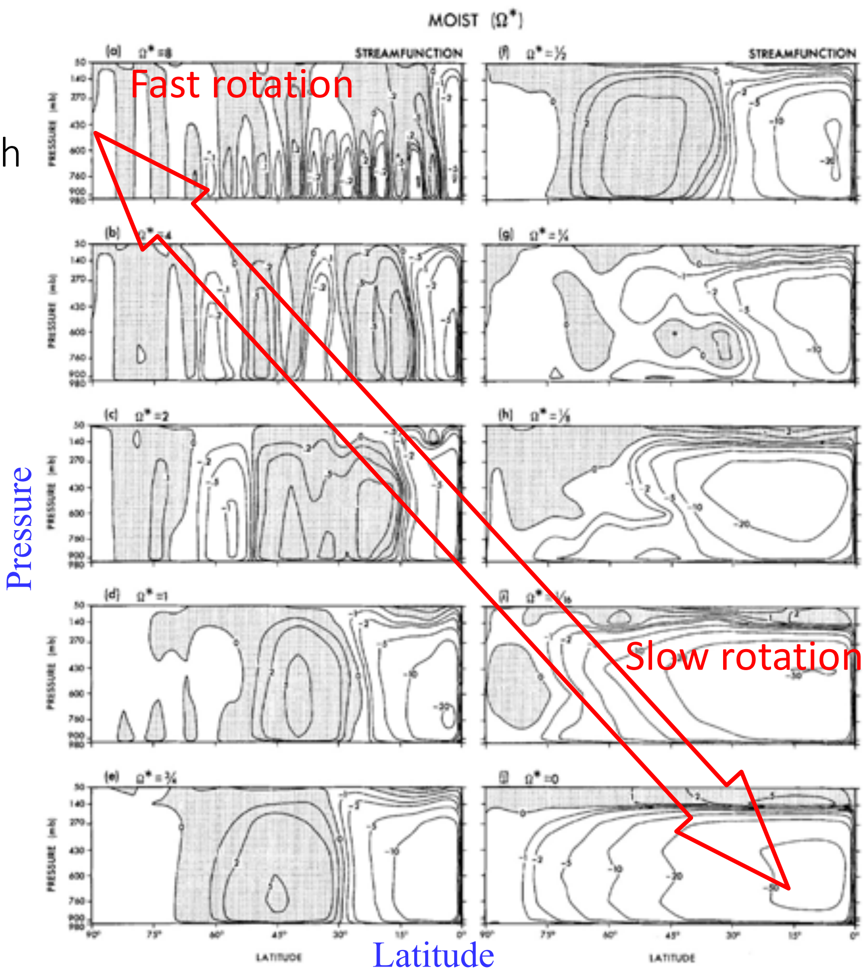
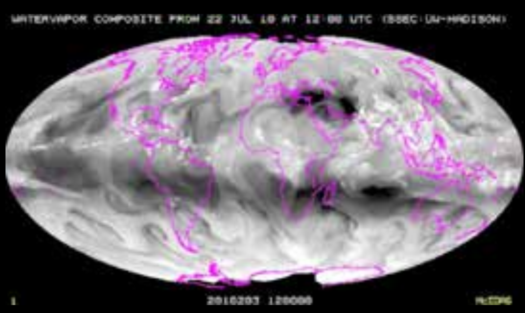
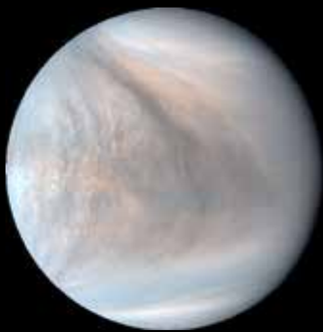


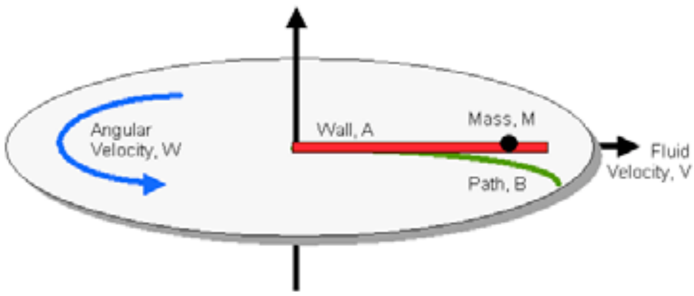
Fig. 3. Meridional distribution of the mean stream function for the MOIST model with $\Omega^* = 0$-8. Units: 10^{13} g s^{-1}





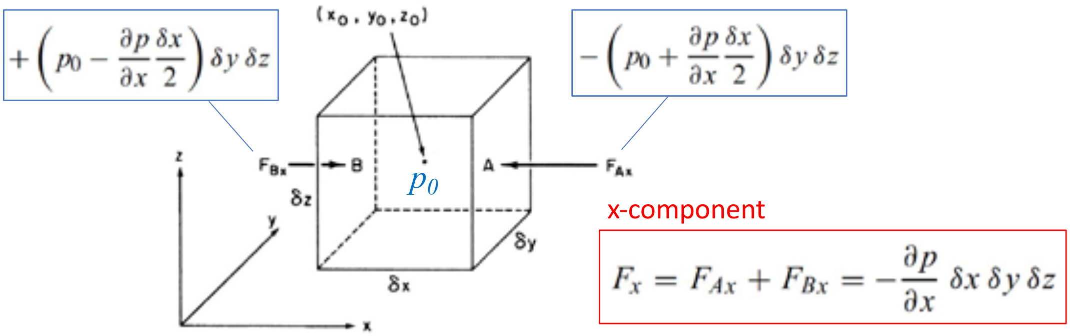
Why do zonal currents predominate ?

Momentum eq. in a rotating frame



Momentum equation

Pressure gradient force:



The x component of the pressure gradient force acting on a fluid element.

Holton, 2004

Momentum equation in the inertial coordinate system

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla p + \vec{g}$$

$\vec{v}$: velocity
t : time
ρ : atmospheric density
p : atmospheric pressure
$\vec{g}$: gravity acceleration

Momentum eq. in a rotating frame

Let us consider a frame rotating at an angular velocity $\vec{\Omega}$.

For a position vector $\vec{r}$, the relationship between the time derivative in the inertial frame d_r/dt and that in a rotating frame d/dt is

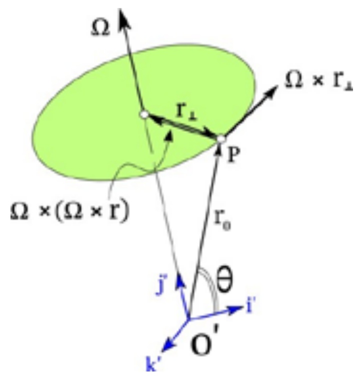
$$\frac{d\vec{r}}{dt} = \frac{d_r\vec{r}}{dt} + \vec{\Omega}\times\vec{r}$$

$$\therefore \vec{v} = \vec{v}_r + \vec{\Omega}\times\vec{r}$$

$\vec{v}_r$: velocity in rotating frame

Similarly

$$\frac{d\vec{v}}{dt} = \frac{d_r\vec{v}}{dt} + \vec{\Omega}\times\vec{v}$$



Combining these, using $\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = -\Omega^2\vec{R}$ we have

$$\frac{d\vec{v}}{dt} = \frac{d_r\vec{v}_r}{dt} + 2\vec{\Omega}\times\vec{v}_r - \Omega^2\vec{R}$$

$\vec{R}$: vector from the rotational axis to the fluid position

Substituting the relationship into the momentum equation, one gets

$$\frac{d_r \vec{v}_r}{dt} = -2\vec{\Omega} \times \vec{v}_r - \frac{1}{\rho}\nabla p + \vec{g} + \Omega^2 \vec{R}$$

Hereafter the subscript r is omitted. The effective gravity acceleration is defined as the sum of $\vec{g}$ and $\Omega^2 \vec{R}$. Then the momentum equation in the rotating frame is

$$\frac{\partial \vec{v}}{\partial t} = -\vec{v} \cdot \nabla \vec{v} - 2\vec{\Omega} \times \vec{v} - \frac{1}{\rho}\nabla p + \vec{g}$$

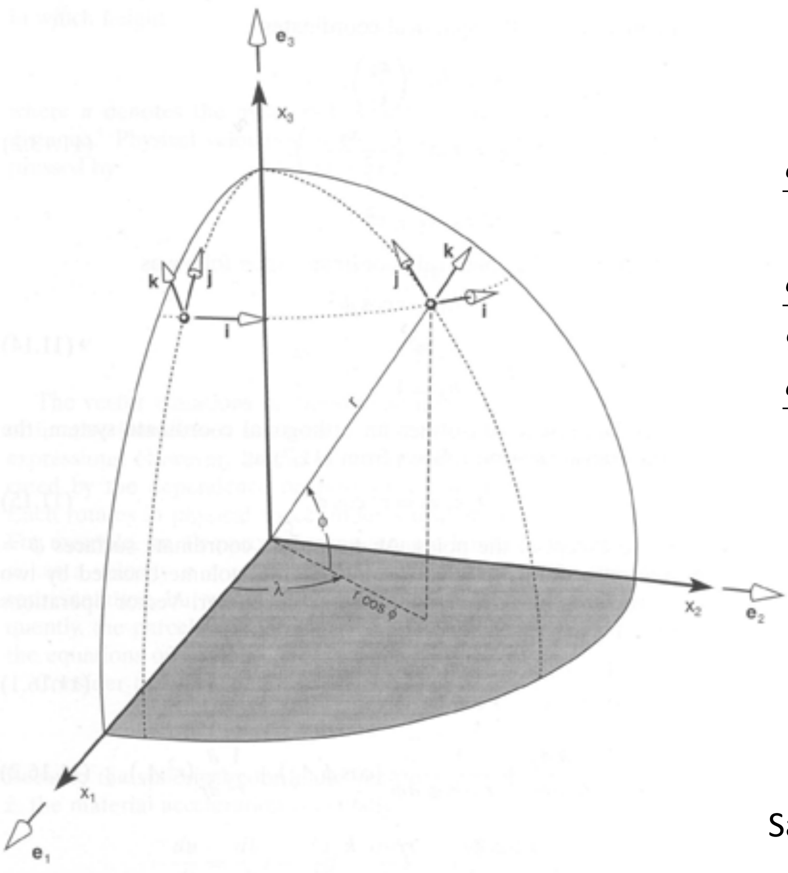
Next we consider an expression in a Cartesian coordinate system on a spherical surface. Using the velocity component (u, v, w) in the unit vector system $(\vec{i}, \vec{j}, \vec{k})$,

$$\vec{v} = \vec{i}u + \vec{j}v + \vec{k}w$$

$\vec{i}$: eastward, $\vec{j}$: northward, $\vec{k}$: upward

Then

$$\frac{d\vec{v}}{dt} = \vec{i}\frac{du}{dt} + \vec{j}\frac{dv}{dt} + \vec{k}\frac{dw}{dt} + u\frac{d\vec{i}}{dt} + v\frac{d\vec{j}}{dt} + w\frac{d\vec{k}}{dt}$$



$$\frac{d\vec{i}}{dt} = u\left(\frac{\tan\phi}{r}\vec{j} - \frac{\vec{k}}{r}\right)$$

$$\frac{d\vec{j}}{dt} = -u\frac{\tan\phi}{r}\vec{i} - \frac{v}{r}\vec{k}$$

$$\frac{d\vec{k}}{dt} = \frac{u}{r}\vec{i} + \frac{v}{r}\vec{j}$$

Salby (1996)

Figure 11.2 Spherical coordinates: longitude λ, latitude ϕ, and radial distance r. Coordinate vectors $\boldsymbol{e}_\lambda = \boldsymbol{i}$, $\boldsymbol{e}_\phi = \boldsymbol{j}$, and $\boldsymbol{e}_r = \boldsymbol{k}$ change with position (e.g., relative to fixed coordinate vectors $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, and $\boldsymbol{e}_3$ of rectangular Cartesian coordinates).

Momentum equations (a: planetary radius)

$$\frac{du}{dt}=\left(\boxed{2\Omega\sin\phi}+\frac{u\tan\phi}{a}\right)v-\frac{uw}{a}-2\Omega w\cos\phi-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{dv}{dt}=-\left(\boxed{2\Omega\sin\phi}+\frac{u\tan\phi}{a}\right)u-\frac{vw}{a}-\frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{dw}{dt}=\frac{u^2+v^2}{a}-2\Omega u\cos\phi-\frac{1}{\rho}\frac{\partial p}{\partial z}-g$$

$f=2\Omega\sin\phi$: Coriolis parameter

The shaded terms are "metric" terms arising from the spherical geometry. These are small on the Earth. (Venus is exception)

When dw/dt = 0 , the equation in the vertical direction reduces to the hydrostatic equilibrium:

$$\frac{\partial p}{\partial z}=-\rho g$$

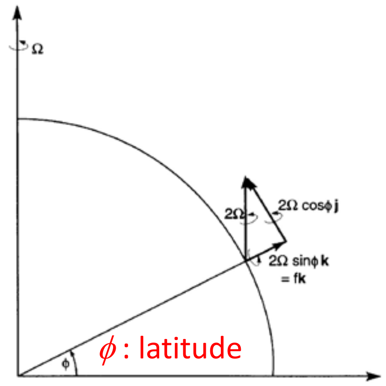


Figure 11.3 Planetary vorticity 2Ω decomposed into horizontal and vertical components.

Neglecting the metric terms and defining $\vec{v}=(u,\,v)$ as the horizontal component of the velocity, we have a set of governing equations:

$$\frac{\partial\vec{v}}{\partial t}=-\vec{v}\cdot\nabla\vec{v}-w\frac{\partial\vec{v}}{\partial z}-f\bar{k}\times\vec{v}-\frac{1}{\rho}\nabla p$$ horizontal momentum eq.

$$\frac{\partial w}{\partial t}=-\vec{v}\cdot\nabla w-w\frac{\partial w}{\partial z}-\frac{1}{\rho}\frac{\partial p}{\partial z}-g$$ vertical momentum eq.

$$\frac{\partial\rho}{\partial t}=-\nabla\cdot(\rho\vec{v})$$ continuity eq.

$$p=\rho RT$$ state eq.

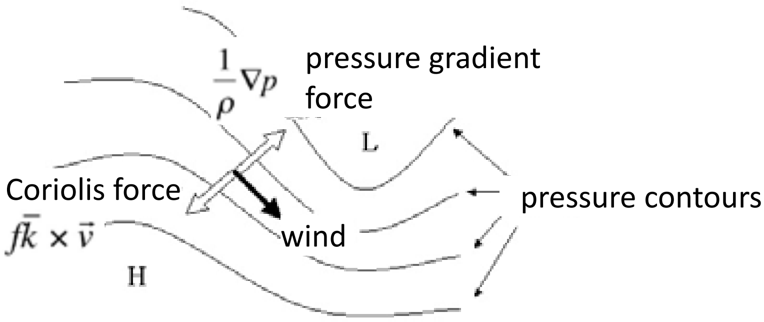
$$\frac{\partial\theta}{\partial t}=-\vec{v}\cdot\nabla\theta-w\frac{\partial\theta}{\partial z}+\underbrace{\frac{1}{c_p}\left(\frac{p_S}{p}\right)^{R/C_p}\dot{H}}_{\text{diabatic heating}}$$ thermodynamics eq.

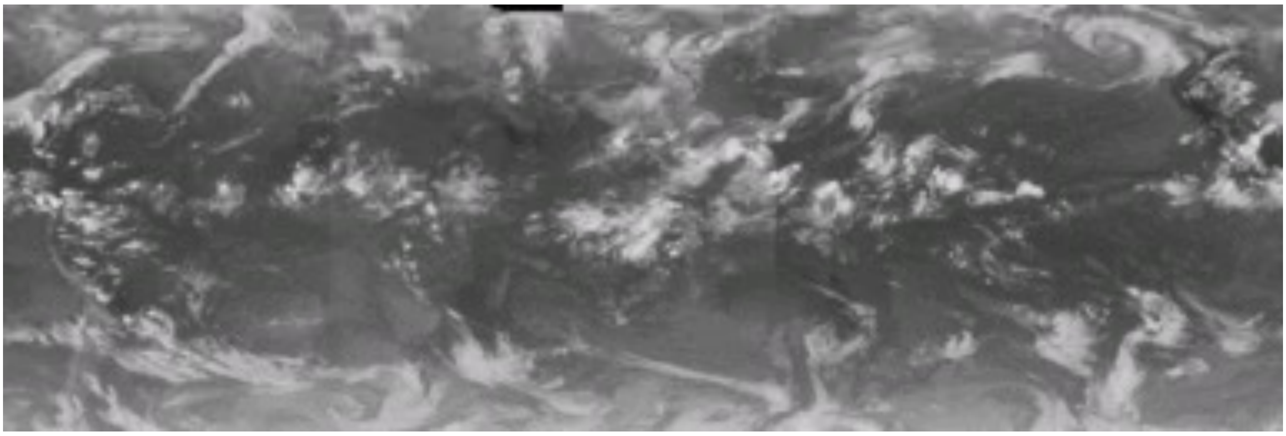
$\vec{k}$: unit vector in the vertical direction

$f=2\Omega\sin\phi$: Coriolis parameter

$\dot{H}$: diabatic heating rate (J/m^3/s)

Geostrophic flow





Geostrophic flow

For synoptic scale (>1000 km) motions in Earth's atmosphere:

- horizontal scale L ~ 1000 km
- pressure scale P ~ 1000 hPa
- velocity scale U ~ 10 m/s
- time scale L/U ~ 10^5 s
- Coriolis parameter f ~ 10^{-4} Hz

$$\cancel{\frac{\partial \vec{v}}{\partial t}} = -\cancel{\vec{v}\cdot\nabla\vec{v}} - \cancel{\omega\frac{\partial \vec{v}}{\partial p}} - \underbrace{f\bar{k}\times\vec{v}}_{\text{Coriolis force}} - \underbrace{\nabla\Phi}_{\text{pressure gradient force}}$$

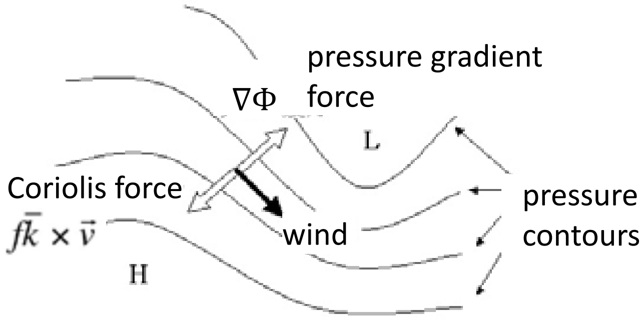
10^{-4} $\quad 10^{-4}$ $\quad 10^{-4}$ $\quad 10^{-3}$ $\quad 10^{-3}$ $\quad (\mathrm{m\ s^{-2}})$

The ratio between the acceleration (U^2/L) and the Coriolis force (fU)

$$R_o \equiv \frac{U}{fL}$$

: Rossby number (around 0.1 on Earth, 10–100 on Venus)

Geostrophic flow



Geostrophic flow becomes a good approximation for small Rossby numbers

$$f\bar{k}\times\vec{v}_g = -\nabla\Phi$$

Definition of geostrophic flow

$$\vec{v}_g = \frac{1}{f}\vec{k}\times\nabla\Phi \qquad \text{or} \qquad u_g = -\frac{1}{f}\frac{\partial\Phi}{\partial y},\quad v_g = \frac{1}{f}\frac{\partial\Phi}{\partial x}$$

Geostrophic flow is two-dimensional:

$$\nabla\cdot\vec{v}_g = 0$$
$$\omega = 0$$

Geostrophic flow is not a good approximation near the equator (*f* is small) or for scales < O(100km) (*L* is small) where Ro (= U/fL) is large.

Pressure coordinate

Large-scale atmospheric motions satisfy hydrostatic equilibrium.
In this case the pressure *p* can be used as the vertical coordinate:

$$p = p(z) \quad \rightarrow \quad \boxed{z = z(p)}$$

p : pressure, *z* : altitude

$\boxed{\omega} \equiv dp/dt$: vertical velocity in pressure coordinate

$\boxed{\Phi}$: geopotential $\quad d\Phi = gdz$

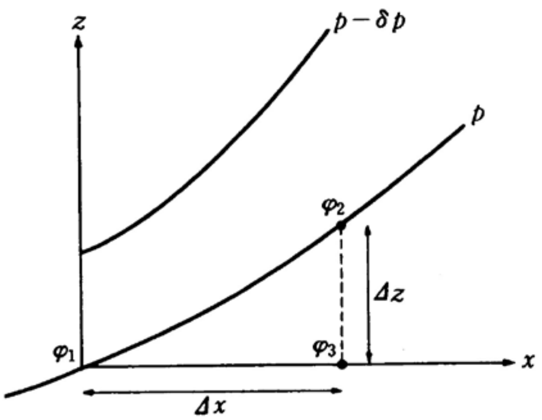


図 3.2 x–z 面内の断面.

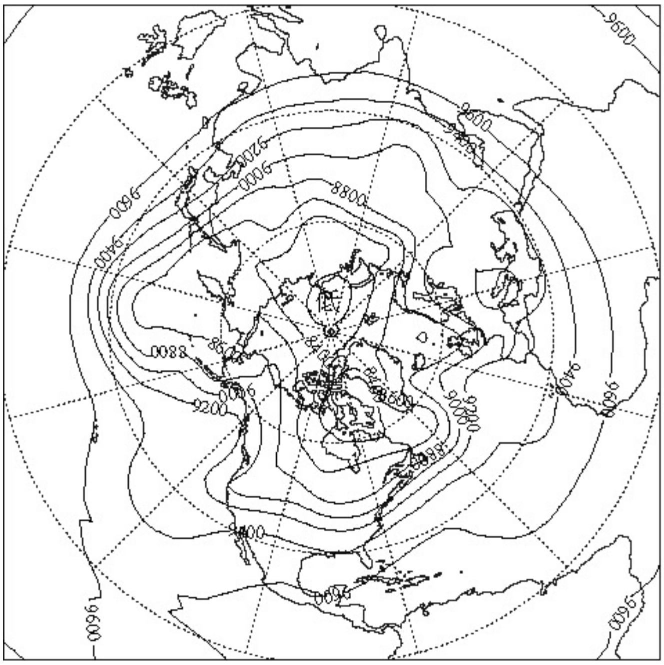
小倉(1978)

Approximate relationship between ω and *w*:

$$\omega \equiv dp/dt$$
$$\sim -\, w/\rho g$$

Pressure coordinate

300hPa, 2002/1/15



Pressure coordinate

“Primitive equations”

$$\frac{\partial \vec{v}}{\partial t} = -\vec{v}\cdot\nabla\vec{v} - \underline{\omega\frac{\partial \vec{v}}{\partial p}} - f\vec{k}\times\vec{v} - \underline{\nabla\Phi}$$ horizontal momentum eq.

$$\underline{\frac{\partial \Phi}{\partial p} = -\frac{RT}{p}}$$ state eq. + hydrostatic equilibrium

$$\nabla\cdot\vec{v} + \underline{\frac{\partial \omega}{\partial p}} = 0$$ continuity eq.

$$\frac{\partial \theta}{\partial t} = -\vec{v}\cdot\nabla\theta - \underline{\omega\frac{\partial \theta}{\partial p}} + \frac{1}{c_p}\left(\frac{p_S}{p}\right)^{R/C_p}\dot{H}$$ thermodynamics eq.

$\vec{v} = (u, v)$: horizontal velocity

ω : vertical velocity

$\vec{k}$: unit vector in the vertical direction

$f = 2\Omega\sin\phi$: Coriolis parameter

$\dot{H}$: diabatic heating rate (J/m^3/s)

Thermal wind

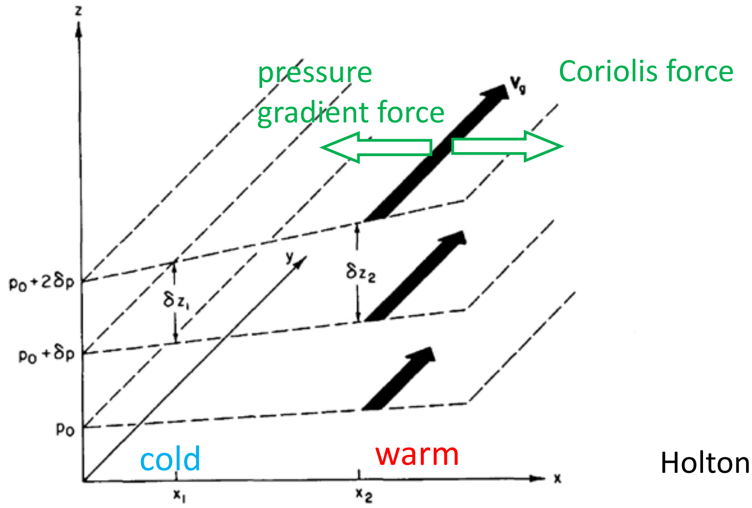
Differentiating $f\bar{k} \times \vec{v}_g = -\nabla\Phi$ with respect to *p* and using $\dfrac{\partial\Phi}{\partial p} = -\dfrac{RT}{p}$,
“thermal wind“ relation is obtained:

$$\frac{\partial \vec{v}_g}{\partial p} = -\frac{R}{fp}\vec{k} \times \nabla T$$

or

$$\frac{\partial u_g}{\partial p} = \frac{R}{fp}\frac{\partial T}{\partial y}$$

$$\frac{\partial v_g}{\partial p} = -\frac{R}{fp}\frac{\partial T}{\partial x}$$

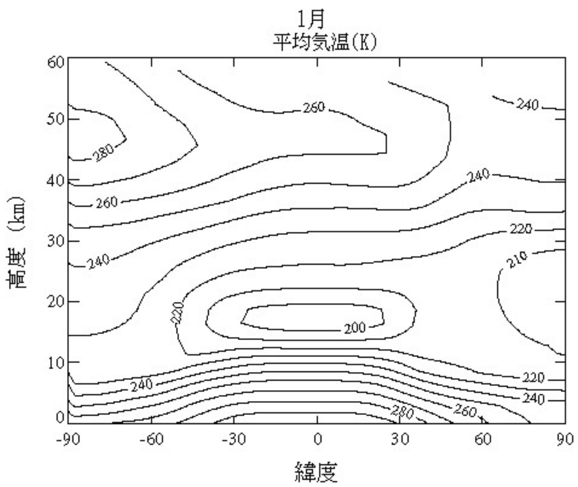


Holton (2004)

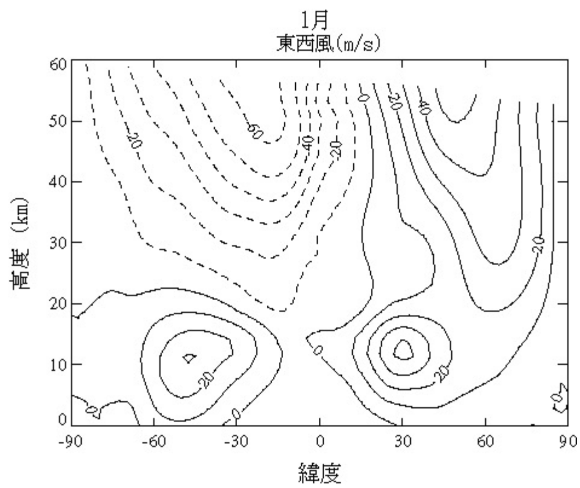
Fig. 3.8 Relationship between vertical shear of the geostrophic wind and horizontal temperature gradients. (Note: $\delta p < 0$.)

Latitude-altitude cross section of Earth’s atmosphere

Temperature



Eastward wind



Latitude-altitude cross section of Mars's atmosphere

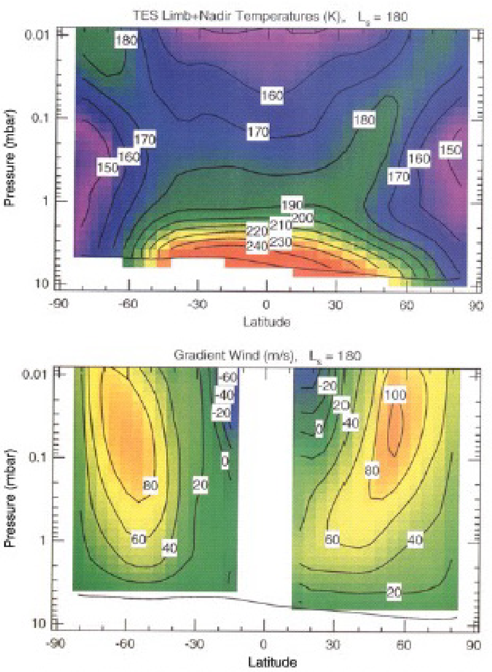


Plate 6. Mean meridional cross sections of (top) retrieved atmospheric temperatures and (bottom) gradient wind speeds for $L_s = 180°$ (northern hemisphere fall equinox). Temperatures are in kelvins, and wind speeds are in m s^{-1}. Positive wind speeds are eastward. The mean surface pressures for this data set are indicated at the lower boundary of the contoured regimes.

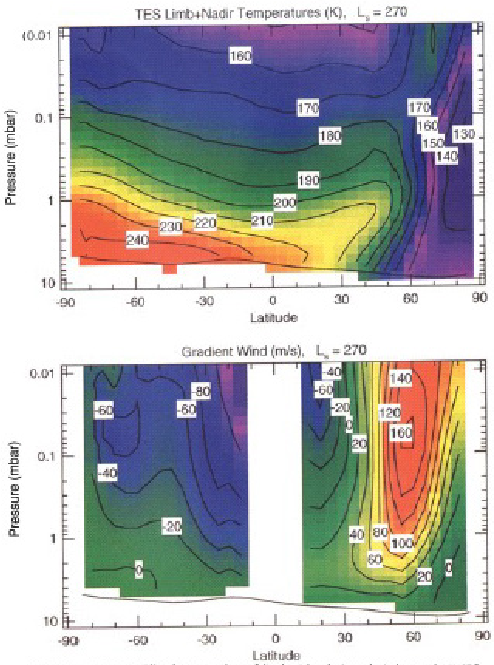


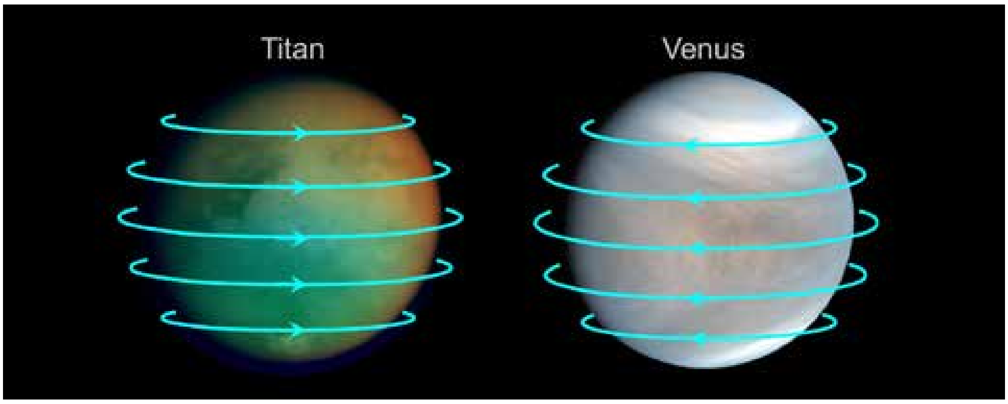
Plate 7. Mean meridional cross sections of (top) retrieved atmospheric temperatures and (bottom) gradient wind speeds for $L_s = 270°$ (northern winter solstice). Temperatures are in kelvins, and wind speeds are in m s^{-1}. Positive wind speeds are eastward. The mean surface pressures for this data set are indicated at the lower boundary of the contoured regimes.

Temperature

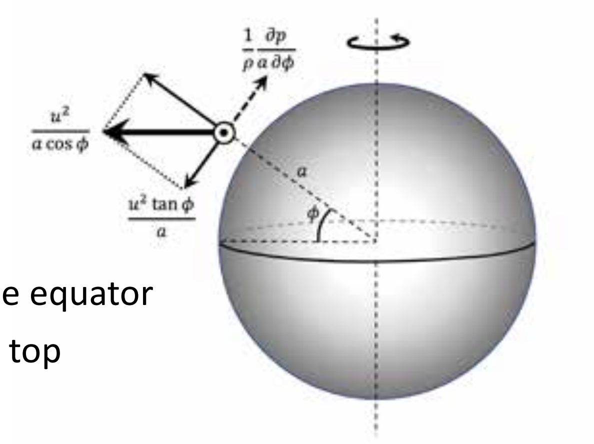
Eastward wind

(Smith et al. 2001)

Superrotation



Superrotation



Prevails in Venus's atmosphere

planetary rotation: 1.8 m/s on the equator

zonal wind: 100 m/s at the cloud top

Momentum equations

$$\frac{du}{dt} = \left(2\Omega\sin\phi + \boxed{\frac{u\tan\phi}{a}}\right)v - \frac{uw}{a} - 2\Omega w\cos\phi - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{dv}{dt} = -\left(2\Omega\sin\phi + \boxed{\frac{u\tan\phi}{a}}\right)u - \frac{vw}{a} - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

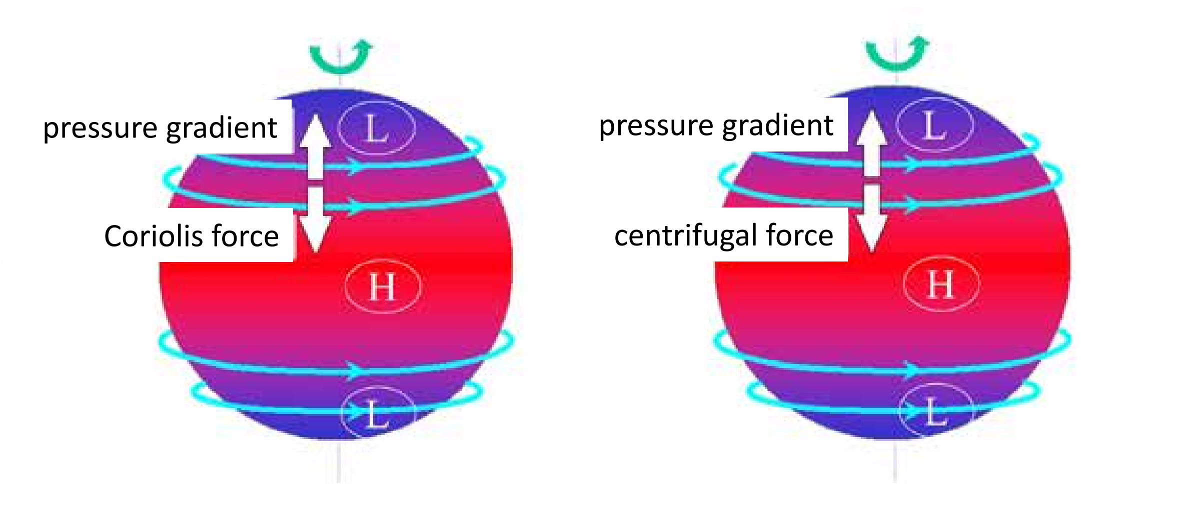
$$\frac{dw}{dt} = \frac{u^2+v^2}{a} - 2\Omega u\cos\phi - \frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Atmospheric rotation takes the place of planetary rotation.

Meridional force balance of zonal flow

geostrophic flow
(planetary rotation >> wind)

cyclostrophic flow
(planetary rotation << wind)



Earth-like

Venus-like

Thermal wind

geostrophic flow
(planetary rotation >> wind)

cyclostrophic flow
(planetary rotation << wind)

$$\cancel{\frac{dv}{dt}} = -\left(\boxed{2\Omega\sin\phi} + \boxed{\frac{u\tan\phi}{a}}\right)u - \cancel{\frac{vw}{a}} - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\boxed{2\Omega\sin\phi\, a\, u + \frac{1}{\rho}\frac{\partial p}{\partial \phi} = 0}$$

$$\boxed{u^2\tan\phi + \frac{1}{\rho}\frac{\partial p}{\partial \phi} = 0}$$

← state eq.
← hydrostatic equilibrium

$$\boxed{\frac{\partial u}{\partial \zeta} + \frac{R}{2a\Omega\sin\phi}\frac{\partial T}{\partial \phi} = 0}$$

$$\boxed{\frac{\partial u^2}{\partial \zeta} + \frac{R}{\tan\phi}\frac{\partial T}{\partial \phi} = 0}$$

$\zeta = -\ln p$: log-pressure altitude

Latitude-altitude cross section of Venus's atmosphere

Temperature

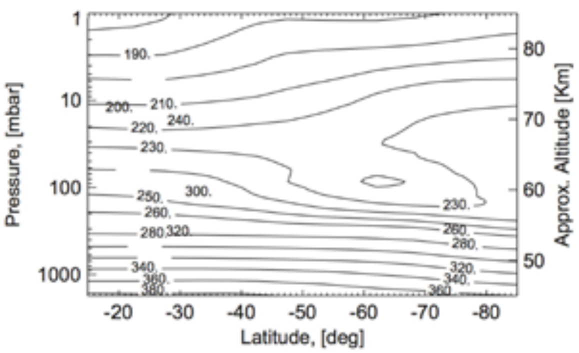


Fig. 2. Meridional cross section of atmospheric temperature (K) obtained combining 116 VeRa profiles. Hemispherical symmetry and local time independence has been assumed. Contours have been smoothed. Contours interval is 10 K, some contours have been removed to render the plot clearer.

Westward wind

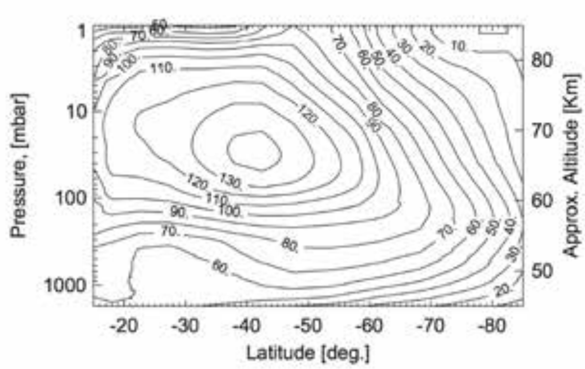
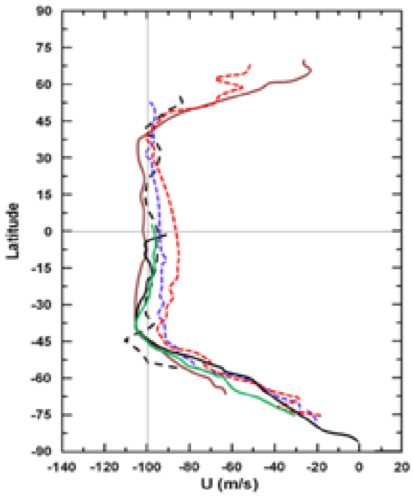


Fig. 8. Latitude–height cross section of zonal thermal wind speed ($m\,s^{-1}$) derived from VeRa temperature profiles assuming cyclostrophic balance (Eq. (5)). The velocity used as lower boundary condition is the cloud-tracked wind profile retrieved from VIRTIS/VEx 1.74 μm images (Sánchez-Lavega et al., 2008). Contour interval is 10 $m\,s^{-1}$.

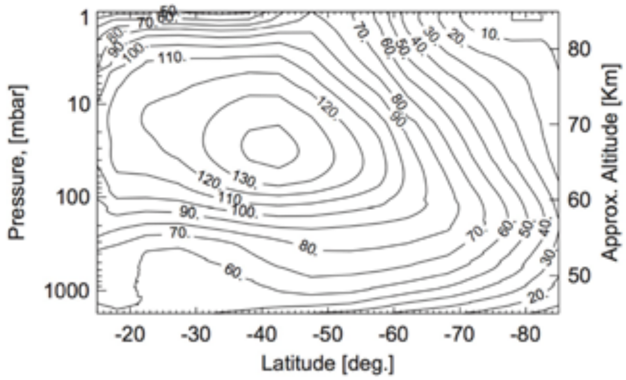
(Piccialli et al., 2012)

Latitude-altitude cross section of Venus's atmosphere

Cloud–tracked zonal winds around 70 km (Sánchez–Lavega et al. 2017)



Zonal winds determined from cyclostrophic balance (Piccialli et al., 2012)



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The discrepancy is not understood

Radiative energy budget and meridional circulation

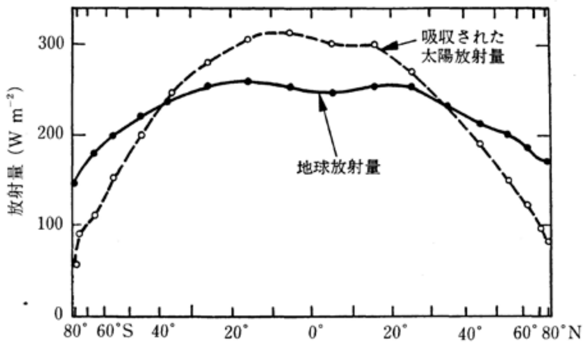


図 4. 4　1962〜66 年の衛星観測に基づく帯状平均した放射収支の緯度分布（Vonder Haar and V. Suomi, 1971 : *J. Atmos. Sci.*, 28, 305-314.）

小倉(1978)

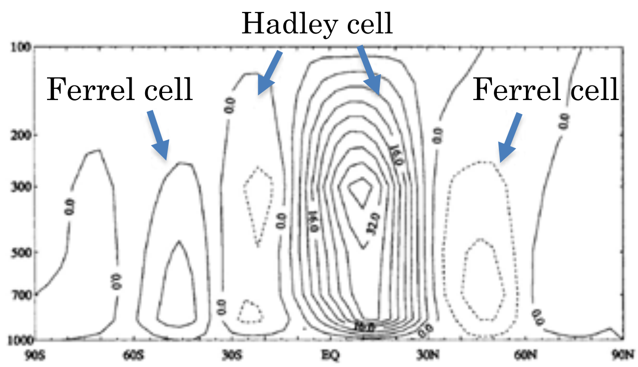


Fig. 10.7 Streamfunction (units: 10^2 kg m^{-1}s^{-1}) for the observed Eulerian mean meridional circulation for Northern Hemisphere winter, based on the data of Schubert et al. (1990).

Holton (2004)

Horizontal convection →Hadley cell

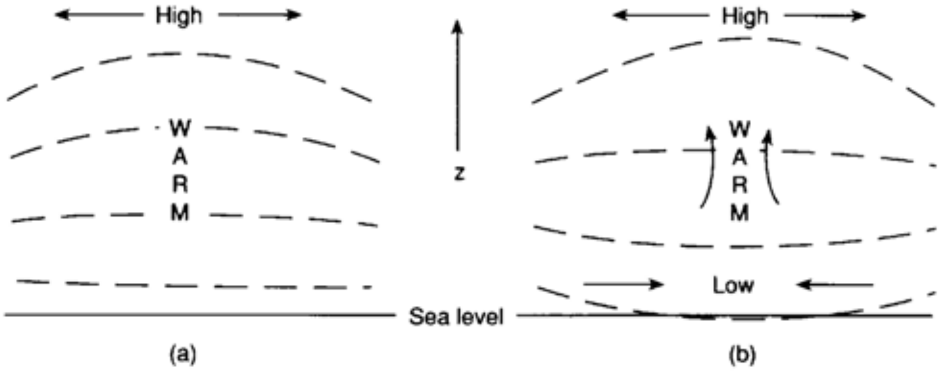
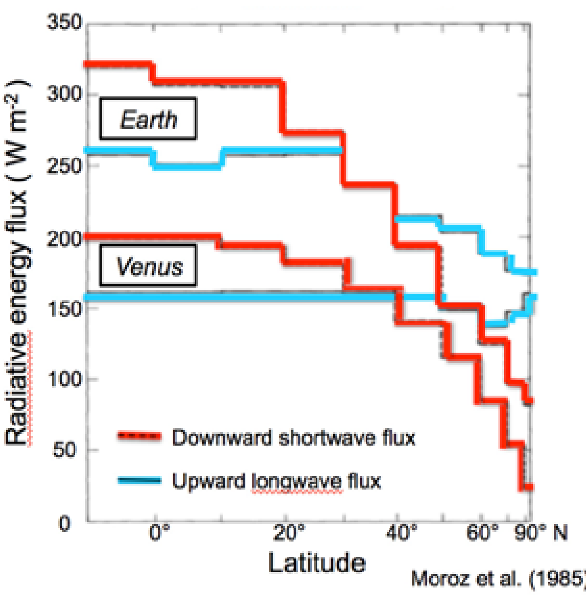


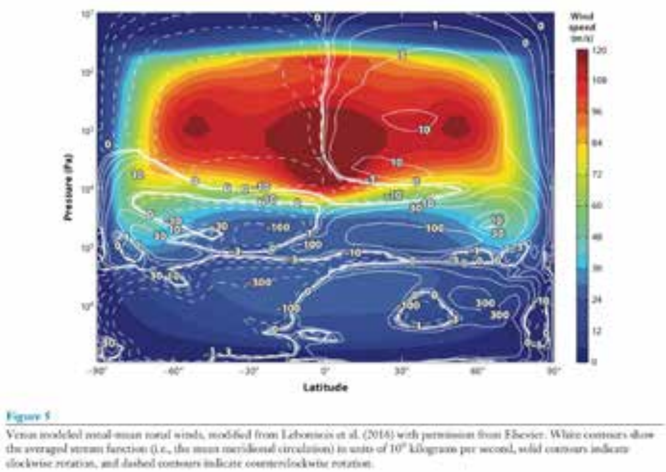
Fig. 3.11 Adjustment of surface pressure to midtropospheric heat source. Dashed lines indicate isobars.

Holton (1992)

Radiative energy budget and meridional circulation



Moroz et al. (1985)

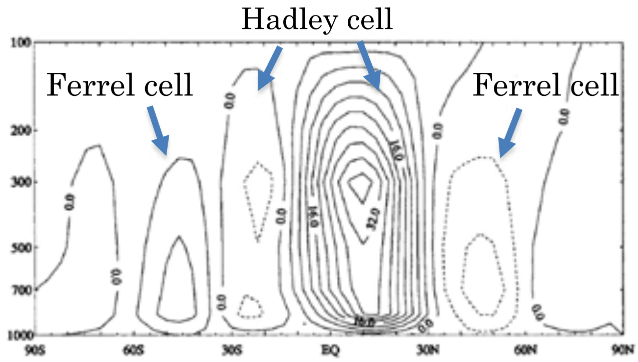


Read & Lebonnois (2018)

Mid-latitude disturbances

3-D disturbance arising from instability of zonal jets
(baroclinic instability) → Ferrel cell

（伊藤 1976）



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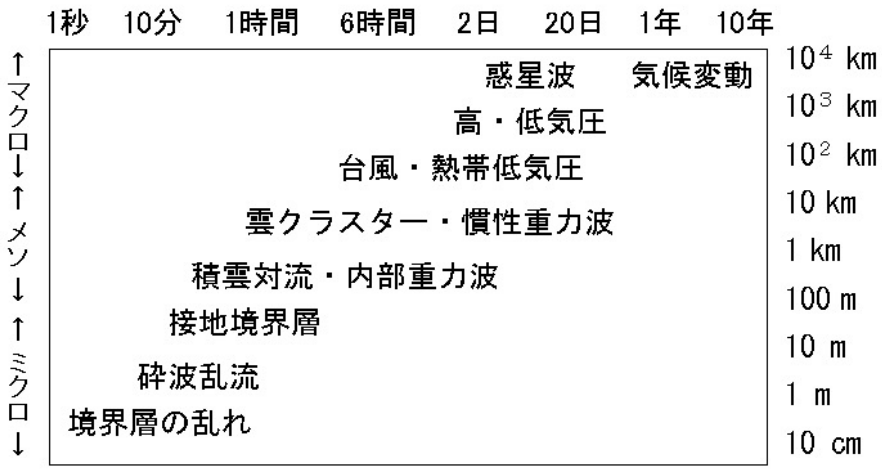
Holton (2004)

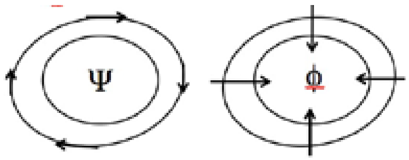
Hierarchical structure of Earth's meteorology



Rotational wind and divergent wind

The horizontal velocity vector can be expressed with the stream function ψ and the velocity potential ϕ :

$$\vec{v} = \underbrace{\vec{k} \times \nabla\psi}_{\text{Rotational wind}} + \underbrace{\nabla\phi}_{\text{Divergent wind}}$$



Rotational component is nodivergent

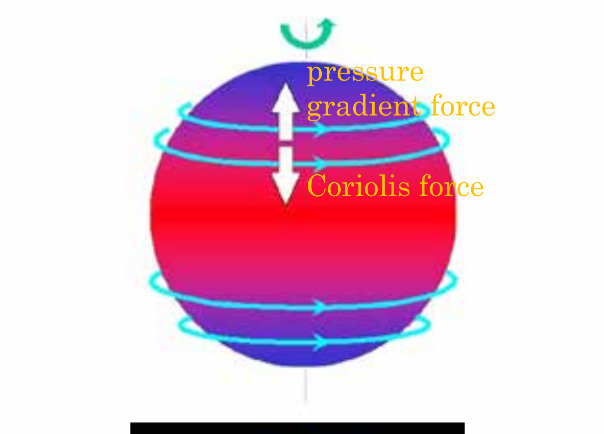
$\nabla \cdot (\vec{k} \times \nabla\psi) = 0$: geostrophic flow, Rossby wave

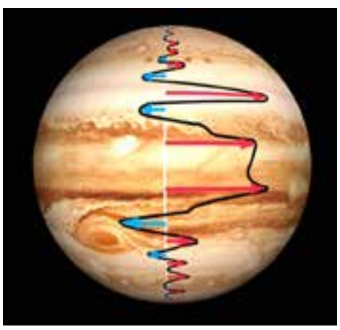
Divergent component is irrotational

$\nabla \times \nabla\phi = 0$: convection, gravity wave

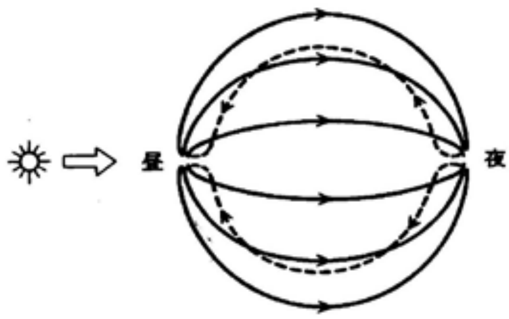
Planetary-scale motions

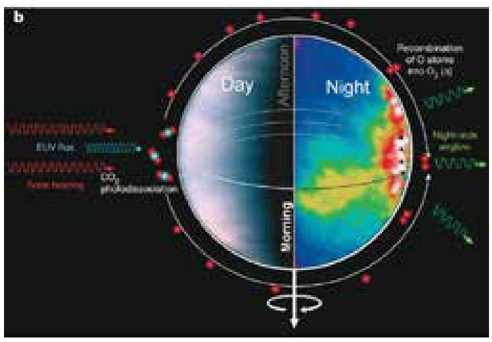
mean zonal wind (rotational flow)

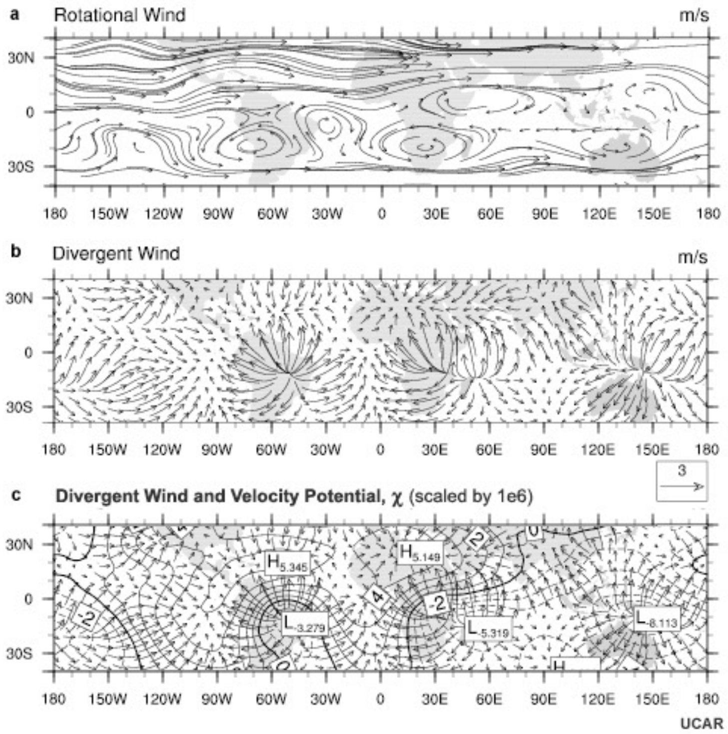




subsolar-to-antisolar circulation (divergent flow)



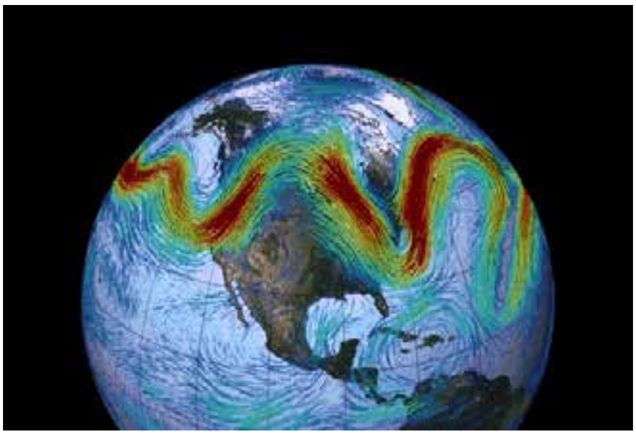




Atmospheric waves

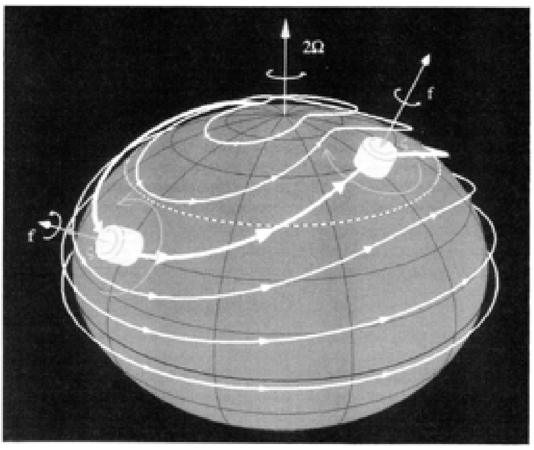
- generated in an unstable background atmosphere
- can transport momentum and energy over long distances
- can induce mixing of the atmosphere

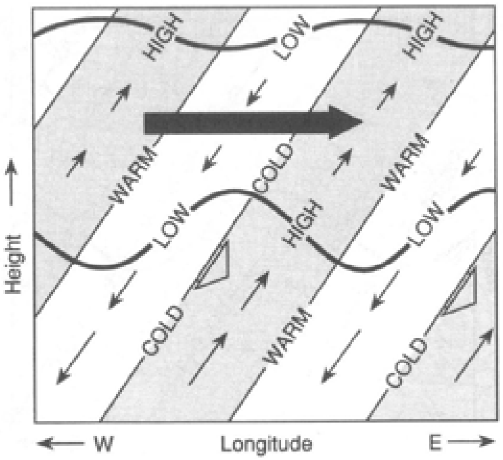
→ Waves play crucial roles in the development of planetary-scale atmospheric structure



Atmospheric waves

Rossby wave (horizontal oscillation)　　Gravity wave（vertical oscillation）





Salby (1996)

Vorticity

The *circulation C* about a closed contour is defined as the line integral along the contour of the component of the velocity vector that is locally tangent to the contour:

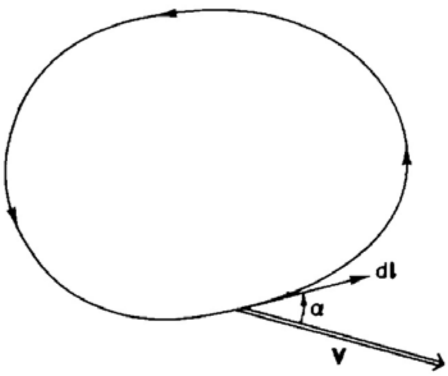
$$C = \oint_l \vec{v} \cdot d\vec{l} = \oint_l |\vec{v}| \cos \alpha dl = \iint_S (\nabla \times \vec{v}) \cdot \vec{n} dS$$

n : unit vector normal to the surface

(Stokes' theorem was used.)

Dividing the circulation δC encircling a small area δS, and considering the limit $\delta S \rightarrow 0$, we get the *vorticity*:

$$\zeta = \frac{\delta C}{\delta S} = (\nabla \times \vec{v}) \cdot \vec{k}$$
$$= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$



Holton (1992)

Quasi-geostrophic approximation

Even in a small Rossby number regime, non-geostrophic (ageostrophic) components play crucial roles in driving vertical winds and temporal changes.

The real wind is divided into geostrophic wind and ageostrophic wind.

$$\vec{v} = \vec{v}_g + \vec{v}_a$$

$$\vec{v}_g \equiv \frac{1}{f_0}\vec{k} \times \nabla\Phi$$

$$R_o \equiv \frac{U}{fL}$$

$$\frac{v_a}{v_g} = O(R_o)$$

The ratio of the magnitudes of the ageostrophic and geostrophic winds is the same order of magnitude as the Rossby number (around 0.1 in Earth's atmosphere)

Lagrangian derivative: $$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\nabla + \omega\frac{\partial}{\partial p}$$

The advection term is approximated by advection by geostrophic wind:

$$\frac{d_g}{dt} \equiv \frac{\partial}{\partial t} + \boxed{\vec{v}_g}\cdot\nabla$$

Beta-plane approximation

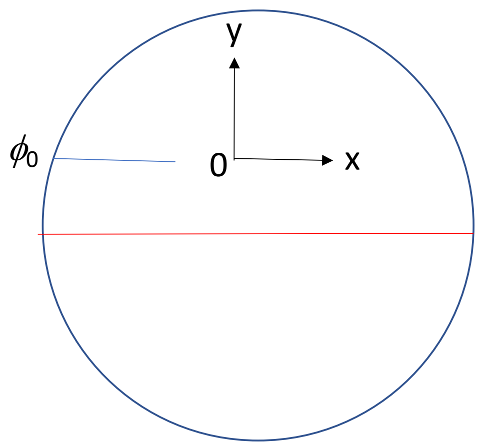
The first-order Taylor series approximation of the Coriolis parameter f :

$$\begin{aligned} f &= 2\Omega\sin\phi \\ &\sim 2\Omega\,[\sin\phi_0 + (\phi-\phi_0)\cos\phi_0] \\ &= \boxed{f_0 + \beta y} \end{aligned}$$

where

$$f_0 = 2\Omega\sin\phi_0$$

$$\beta = \frac{2\Omega}{a}\cos\phi_0 \sim \frac{df}{dy}$$



Mid-latitude beta-plane : $f = f_0 + \beta y$

Equatorial beta-plane : $f = \beta y$

Quasi-geostrophic vorticity equation

Introducing beta-plane approximation to the quasi-geostrophic Lagrangian derivative and retaining small quantities to the first order, the rate of change of the geostrophic wind is given by

$$\left(\frac{\partial}{\partial t} + \vec{v}_g \cdot \nabla\right)\vec{v}_g = -(f_0 + \beta y)\vec{k} \times (\vec{v}_g + \vec{v}_a) - \nabla\Phi$$

$$\sim -f_0\vec{k} \times \vec{v}_a - \beta y\vec{k} \times \vec{v}_g \qquad (1)$$

($\vec{v}_g \equiv \dfrac{1}{f_0}\vec{k} \times \nabla\Phi$ was used)

Since geostrophic wind is nondivergent ($\nabla\cdot \vec{v}_g = 0$), the continuity eq. is

$$\nabla\cdot \vec{v}_a + \frac{\partial \omega}{\partial p} = 0 \qquad (2)$$

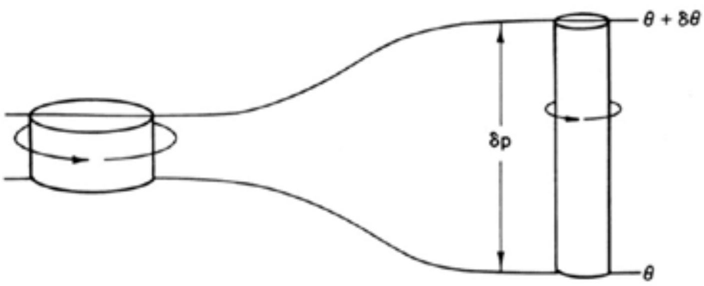
Operating rotation (∇×) to (1) and using (2), we have the quasi-geostrophic vorticity equation.

Quasi-geostrophic vorticity equation

$$\boxed{\frac{\partial \zeta_g}{\partial t} = -\vec{v}_g \cdot \nabla(\zeta_g + f) + f_0\frac{\partial \omega}{\partial p}}$$

$$\zeta_g \equiv \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} = \frac{\nabla^2\Phi'}{f_0}$$: geostrophic vorticity

Vorticity changes with time through
- advection of absolute vorticity ($\varsigma_g + f$) by geostrophic wind ($\vec{v}_g$)
- vertical divergence (horizontal divergence)



Holton (2004)

Fig. 4.7 A cylindrical column of air moving adiabatically, conserving potential vorticity.

Rossby wave

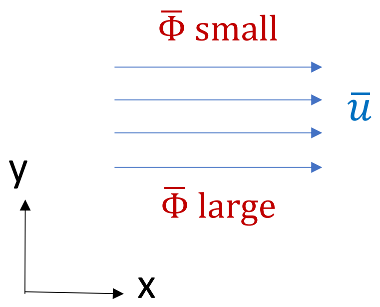
Let us consider a two-dimensional motion ($\omega = 0$)

$$\left(\frac{\partial}{\partial t} + \vec{v}_g \cdot \nabla\right)(\zeta_g + f) = 0$$

→ Absolute vorticity ($\varsigma_g + f$) is conserved along the geostrophic wind $\vec{v}_g$.

A basic state where a homogeneous zonal flow exists:

$$f\overline{u} = -\partial\overline{\Phi}/\partial y$$
$$\overline{\zeta}_g = 0$$



Rossby wave

The deviation from the basic state is denoted by ()' :

$$\frac{\partial}{\partial t}\zeta_g + (\overline{u} + u'_g)\frac{\partial \zeta_g}{\partial x} + v'_g \frac{\partial \zeta_g}{\partial y} + v'_g \beta = 0 \qquad (1)$$

The velocity and vorticity are related to the geopotential perturbation

$$\zeta_g = \frac{\nabla^2 \Phi'}{f_0}, \quad u'_g = -\frac{1}{f_0}\frac{\partial \Phi'}{\partial y}, \quad v'_g = \frac{1}{f_0}\frac{\partial \Phi'}{\partial x}$$

Substituting these into (1) and retaining first order terms only, we get

$$\frac{\partial}{\partial t}\nabla^2\Phi' + \overline{u}\frac{\partial}{\partial x}\nabla^2\Phi' + \beta\frac{\partial \Phi'}{\partial x} = 0$$

Rossby wave

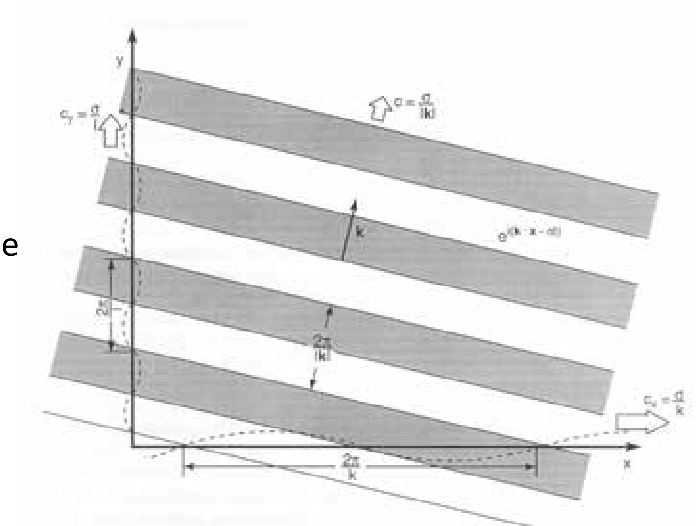
Assuming a wave solution $\Phi' = \hat{\Phi}\exp[i(kx + ly - kct)]$, the phase velocity of Rossby wave is obtained:

$$c - \overline{u} = -\frac{\beta}{k^2 + l^2}$$

k: zonal wavenumber
l: meridional wavenumber
c: zonal phase velocity

- Propagation opposite to the planetary rotation
- β effect (latitude variation of the Coriolis parameter f) is needed.
- Longer waves (smaller k) propagate faster.

The wave possesses angular momentum in the direction opposite to the planetary rotation

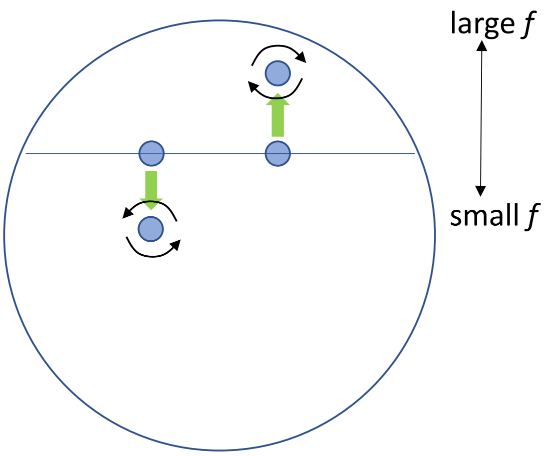


Salby (1996)

Rossby wave

$$\left(\frac{\partial}{\partial t} + \vec{v}_g \cdot \nabla\right)\boxed{(\zeta_g + f)} = 0$$

$$f = 2\Omega \sin\phi$$



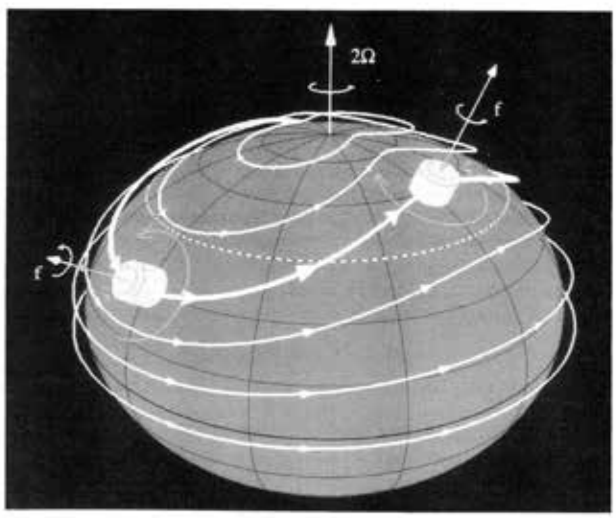
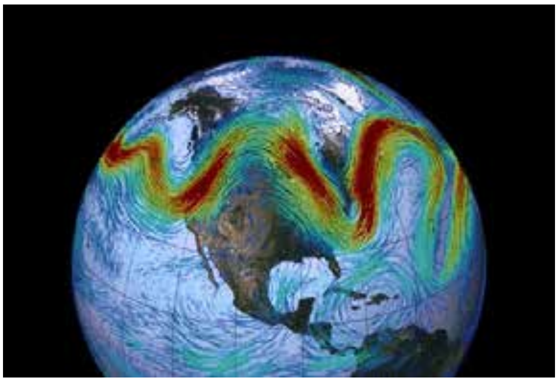
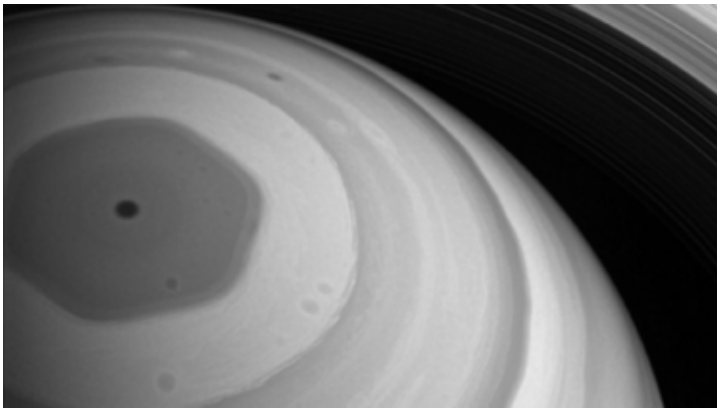


Figure 14.16 Schematic illustrating the reaction of an air parcel to meridional displacement. Displaced equatorward, an eastward-moving parcel spins up cyclonically to conserve absolute vorticity. Northward motion induced ahead of it then deflects the parcel's trajectory poleward back toward its undisturbed latitude. The reverse process occurs when the parcel overshoots and is displaced poleward of its undisturbed latitude.

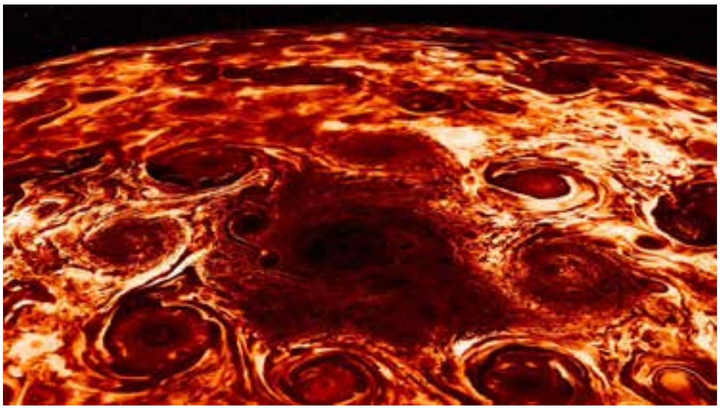
Salby (1996)



Earth

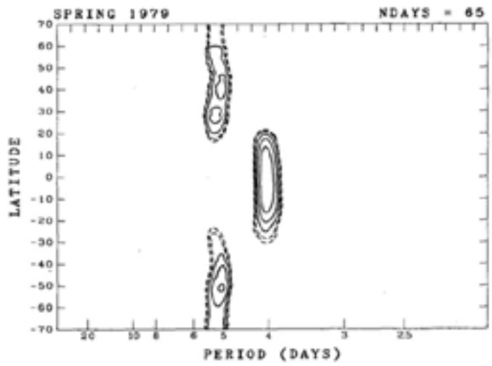


Saturn

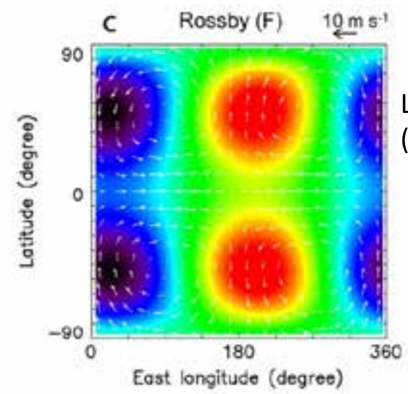


Jupiter

planetary-scale waves on Venus



Power spectra of UV brightness
(DelGenio & Rossow 1990)



Linear solution of Rossby wave at 70 km
(Kouyama et al. 2015)

The superrotation of the atmosphere takes the place of planetary rotation.

Gravity wave

In Cartesian coordinates, without the assumption of hydrostatic equilibrium, the governing equations are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = \frac{1}{\rho}\frac{\partial p}{\partial x}$$

horizontal momentum eq. (x-axis only)

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

vertical momentum eq.

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho w)}{\partial z}$$

continuity eq.

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + w\frac{\partial \theta}{\partial z} = 0$$

thermodynamics eq.

$$\theta = \underbrace{\boxed{\frac{p}{\rho R}}}_{T}\left(\frac{p_S}{p}\right)^{R/C_p}$$

Gravity wave

Equations for disturbances:

$$\bar{\rho}\frac{\partial u'}{\partial t} = -\frac{\partial p'}{\partial x}$$

horizontal momentum eq. (x-axis only)

$$\bar{\rho}\frac{\partial w'}{\partial t} = -\frac{\partial p'}{\partial z} - \rho' g$$

vertical momentum eq.

$$\frac{\partial \rho'}{\partial t} = -\bar{\rho}\frac{\partial u'}{\partial x} - \frac{\partial(\bar{\rho} w')}{\partial z}$$

continuity eq.

$$\frac{1}{\bar{\theta}}\frac{\partial \theta'}{\partial t} + w'\frac{N^2}{g} = 0$$

thermodynamics eq.

$$\rho' = \frac{p'}{c_s^2} - \bar{\rho}\frac{\theta'}{\bar{\theta}}$$

$$c_s^2 = \frac{c_p}{c_v}RT$$

c_s : sound speed

$$N^2 = g\frac{d\ln\bar{\theta}}{dz}$$

N : buoyancy frequency

Gravity wave

Isothermal atmosphere:

$$N^2 = g/c_p T$$

$$\overline{\rho}(z) = \rho_s \exp(-z/H)$$

Substituting the wave solution $w'(x,z,t) = \hat{w}(z)\exp[i(kx + \sigma t)]$ (σ : frequency) into the governing equations before, an equation for the vertical velocity *w* is obtained as

$$\frac{d^2(\overline{\rho}\hat{w})}{dz^2} + \frac{1}{H}\frac{d(\overline{\rho}\hat{w})}{dz} + \left[\frac{\sigma^2}{c_s^2} - k^2 + \frac{N^2 k^2}{\sigma^2}\right]\overline{\rho}\hat{w} = 0$$

Considering the amplitude growth with height in a stratified atmosphere, w is assumed to have the form

$$\hat{w}(z) = W(z)\exp(z/2H) \quad \leftrightarrow \quad \overline{\rho}\hat{w}^2(z) \propto W^2(z)$$

Gravity wave

Then the equation becomes

$$\frac{d^2 W}{dz^2} + \left[\frac{\sigma^2}{c_s^2} - k^2 + \frac{N^2 k^2}{\sigma^2} - \frac{1}{4H^2}\right]W = 0$$

Assuming a wave solution $W \propto \exp(imz)$ (*m*: vertical wavenumber), the dispersion relation is obtained:

$$m^2 = \frac{\sigma^2}{c_s^2} - k^2 + \frac{N^2 k^2}{\sigma^2} - \frac{1}{4H^2}$$

Solutions for acoustic-gravity wave (connected to sound wave) and internal gravity wave exist.

Approximate solution for internal gravity wave is

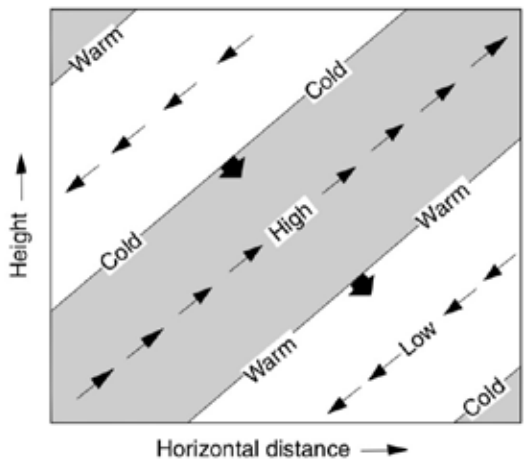
$$\sigma^2 = \frac{N^2 k^2}{k^2 + m^2 + \frac{1}{4H^2}} \qquad \rightarrow \sigma < N$$

Gravity wave

For a large-horizontal scale waves (typical in planetary atmospheres),

$$\frac{k^2}{m^2} = \frac{\sigma^2}{N^2}$$

- Long period waves have near-horizontal phase surfaces



Holton 2004

Fig. 7.9 Idealized cross section showing phases of pressure, temperature, and velocity perturbations for an internal gravity wave. Thin arrows indicate the perturbation velocity field, blunt solid arrows the phase velocity. Shading shows regions of upward motion.

松野・島崎(1981)

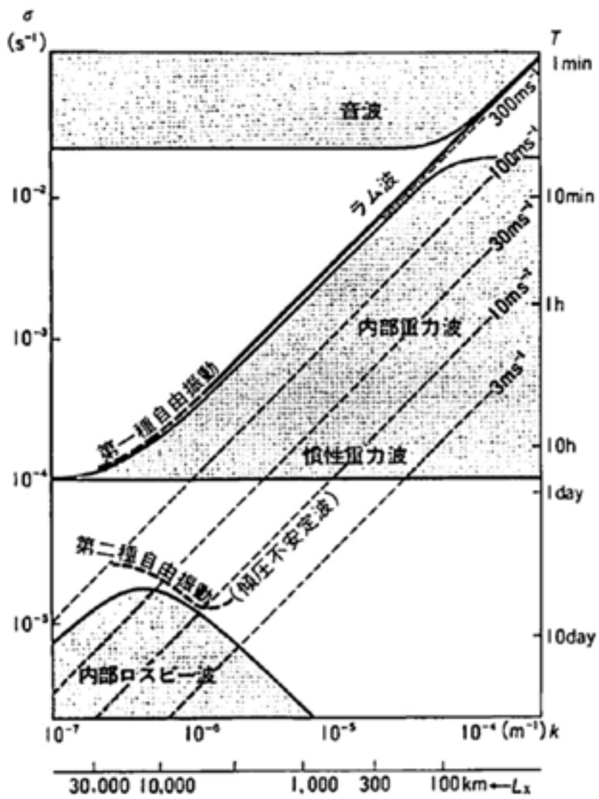
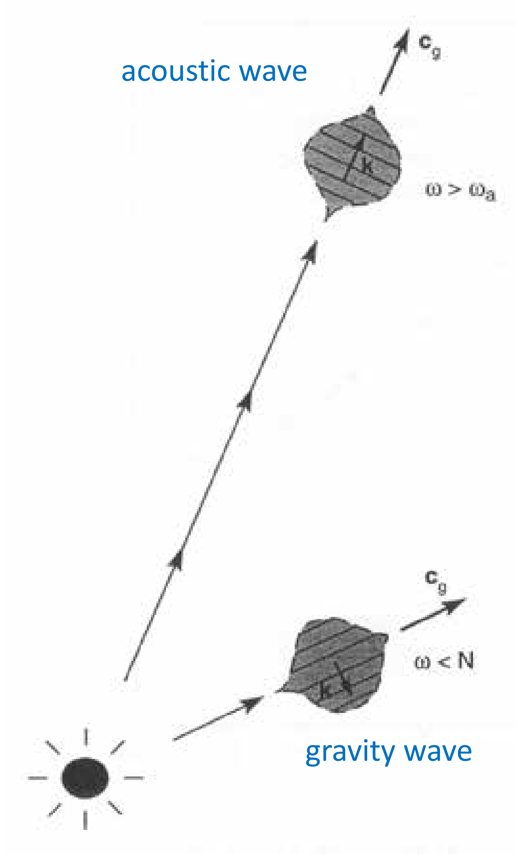
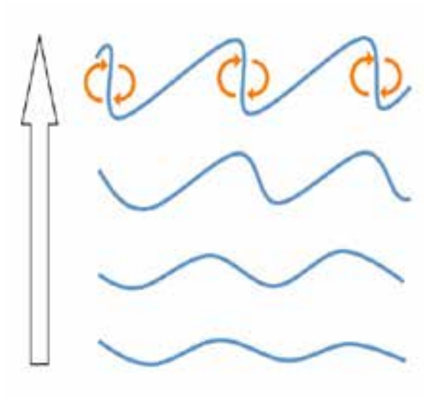


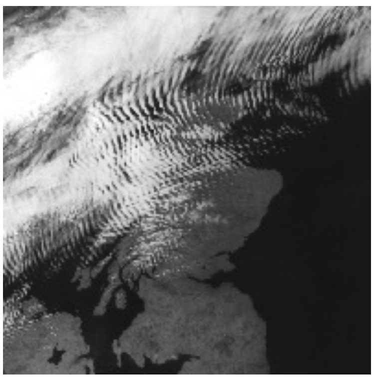
図 5.2 水平波数 k と振動数 σ の関係として各種大気波動の存在域を示す図．斜めの破線は水平位相速度を示す．(k, σ) の組を指定したとき鉛直方向にも波として伝わり得る運動が存在する場合は影がつけてある（内部波領域）．白く残されている部分には三次元的な波動は存在しない（外部波領域）．中・高緯度域に適用できるもので，後で扱う波動もすべて記入してある．大気の自由振動の分散関係も概念的に記入してある．第一種自由振動は波長が短くなるとラム波につながる．第二種自由振動は波長の短い部分で基本流のシアーの影響を受けて傾圧不安定波になると解釈できる．



Salby (1996)



Amplitude growth with height and wave breaking
→ turbulence generation, mean-wind acceleration



Mountain waves on Earth

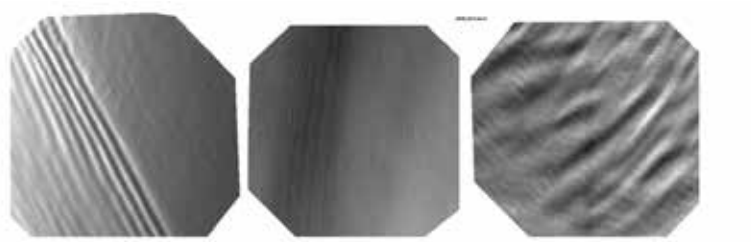
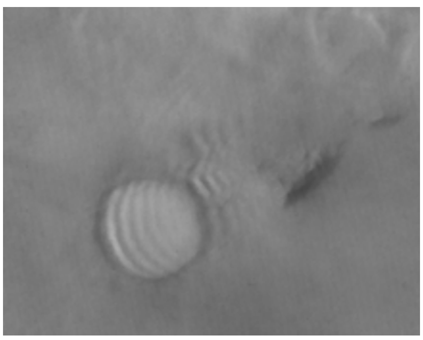
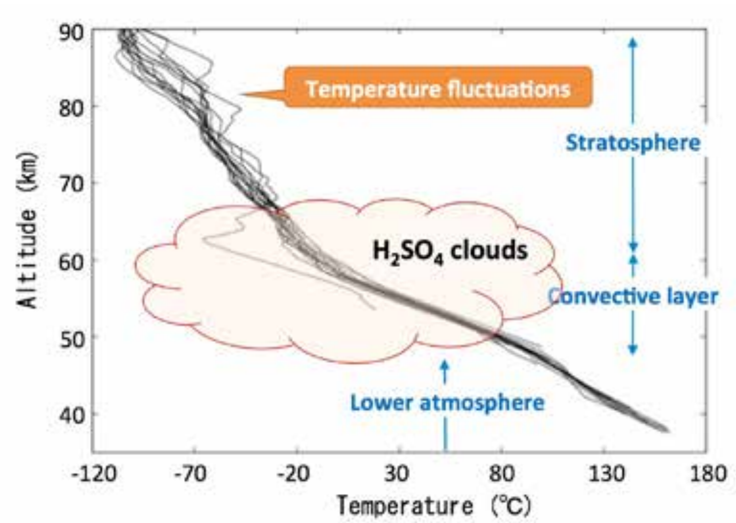


Fig. 11. VMC images of polar waves: left – long waves (NIR) filter), middle – long waves producing short wave trains (UV), right – irregular waves (UV).

on Venus

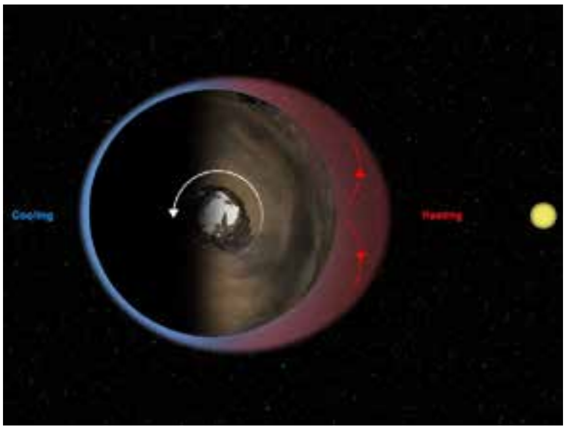


Mountain waves on Mars



Thermal tide

Planetary-scale gravity wave generated by the movement of the solar heating region in the diurnal cycle



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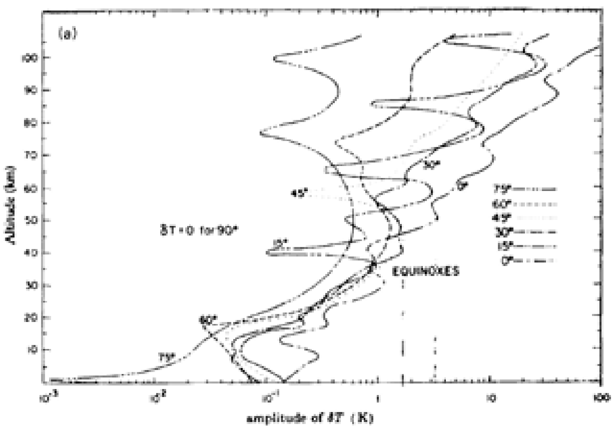
Excitation mechanism:
- Earth : solar heating of stratospheric ozone layer
- Venus : solar heating of cloud layer
- Mars : solar heating of atmospheric dust

Thermal tide in Earth’s atmosphere

Vertical structure



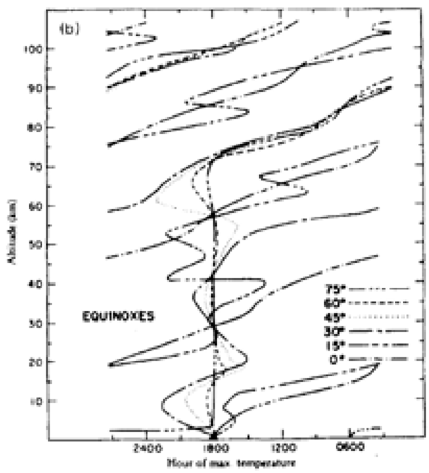
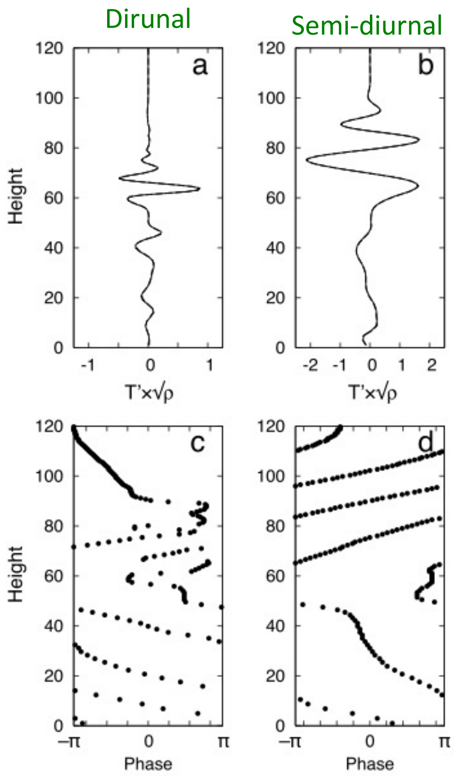


Fig. 4.7. (a) Amplitude and (b) phase of solar diurnal component of T' at various latitudes for equinox. [After Lindzen (1967).]

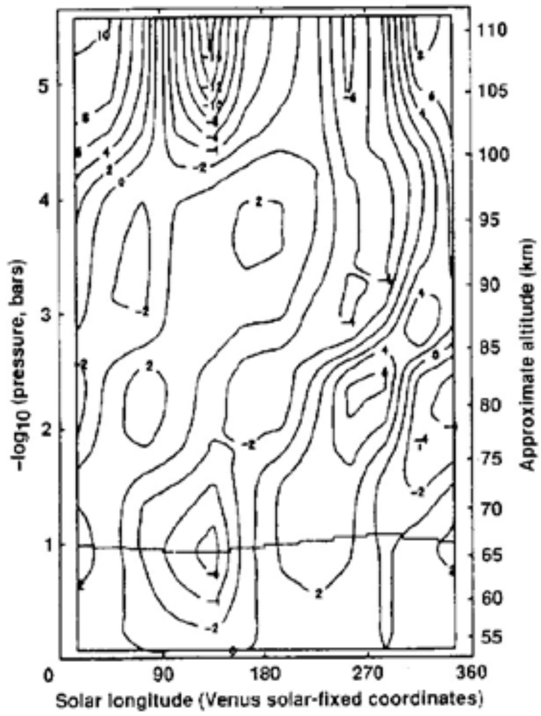
Holton (1992)

Thermal tide in Venus's atmosphere

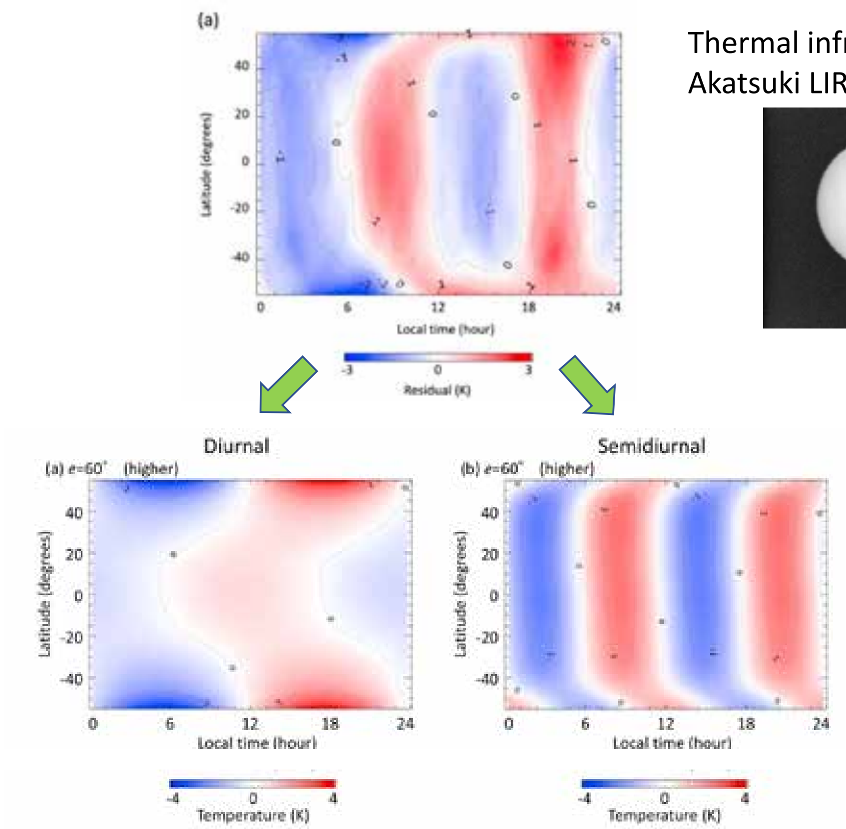
Linear solution (Takagi & Matsuda, 2006)



Temperature perturbation
(Schofield & Taylor 1983)



Thermal tide in Venus's atmosphere



Thermal infrared image taken by Akatsuki LIR